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TESTING AND COMPARING VALUE-AT-RISK MEASURES IN THE BULGARIAN STOCK MARKET

Master’s Thesis in Accounting and Finance

VAASA 2010
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ABSTRACT

The purpose of this thesis is to compare commonly used Value-at-Risk measures calculated through Historical and Monte Carlo Simulations and to answer the question whether these measures adequately capture market risk in EU new member country. Data set of daily returns price for ten years period from 24 October 2000 to 30 April 2010 was collected for the following market indices: SOFIX, S&P 500, NASDAQ, OMXS, FTSE 100 and DAX, to give representative overview of the developed world markets and compare them with the new EU member state Bulgaria. The behaviour of Value-at-Risk models with 99 % and 95 % confidence level using rolling data windows of 100 and 250 days is analyzed with the help of a range of backtesting procedures.

Employed tests revealed that the distribution of daily returns of SOFIX index differs significantly from the Normal distribution, with high kutsosis and large negative skewness. Highest Value-at-Risk violation levels were observed during periods with steep volatility jumps, which indicate that the measure reacts poorly to volatility changes and underestimate risk in turbulent market conditions. Based on the backtesting results it can be derived that VaR models that are commonly used in developed stock markets are not well suited for measuring market risk in EU new member states.

KEYWORDS: Value at risk, Historical Simulation, Monte Carlo Simulation, New EU member states
1 **INTRODUCTION**

The concept of risk and the ways to monitor and control it has drawn a great deal of attention again in the time of economic turbulence and volatile markets. Whenever uncertainty truly unveils its power, financial risk models seems to be caught in a surprise and lag behind in forecasting disastrous outcomes. History of financial cataclysms indicates that extreme events tend to happen much more often then the probabilities given to them by risk models, analysers and managers. In such manner the main purpose of risk management, which is to provide safe net for bad economic conditions, is missed and it looks like “car airbag that works all the time except when you have an accident” (Einhorn 2008: 12).

<table>
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<tr>
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<tr>
<td>Panic of 1907</td>
<td>Year 1907</td>
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<td>Wall Street Crash of 1929</td>
<td>Year 1929</td>
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<tr>
<td>Recession of 1937–1938 (U.S.)</td>
<td>Year 1937</td>
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<td>Silver Thursday</td>
<td>March 27, 1980</td>
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<td>Black Monday</td>
<td>October 19, 1987</td>
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<td>Japanese asset price bubble</td>
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<td>Black Wednesday</td>
<td>September 16, 1992</td>
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<td>1997 Asian Financial Crisis</td>
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<td>1997 mini-crash</td>
<td>October 27, 1997</td>
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<td>1998 Russian financial crisis</td>
<td>August 17, 1998</td>
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<td>dot-com bubble</td>
<td>March 10, 2000</td>
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<td>Sep-11 Terrorist Attacks</td>
<td>September 11, 2001</td>
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<td>Stock market downturn of 2002</td>
<td>Year 2002</td>
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<tr>
<td>Chinese correction</td>
<td>February 27, 2007</td>
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<tr>
<td>Lehman Brothers bankrupt</td>
<td>September 15, 2008</td>
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Looking throughout the history different approaches toward measuring financial risk can be found. Harry Markowitz (1952) proposed mean-variance method for defining volatility in a portfolio. Asset liability management (ALM) model, dated to the high interest rate period in 70’s and 80’s measured discrepancy between debts and assets (Jorion 2000). Jack Treynor (1962), William Shaper (1964) and John Linter (1965)
introduced the Capital asset pricing model (CAPM) that linked the expected rate of return of asset with standard deviation for portfolio diversification purposes.

In the 90’s a group of employees of J.P. Morgan Bank developed the fundamentals of Value at Risk (VaR) methodology. Their endeavour was focused on giving a single money figure expressing possible worst loss, associated with the risk of a derivative, trader’s portfolio or even whole firm, given predefined probability level and time horizon (RiskMetrics 2009). After its public presentation from JPMorgan and the establishment of its Riskmetrics unit as independent consulting company, VaR fast became universal enterprise risk measure. VAR was also implemented by private institutions like the group of thirty (G30), consultative group on international economics and monetary affairs, the global association of risk professionals (GARP) and public regulators-General accounting office (GAO), the financial accounting standards board (FASB), securities and exchange commission (SEC) etc. making VaR the “gold standard” in the risk measurement area. Probably the most significant step was taken when The Basel Committee on Banking Supervision (BCBS) enacted their legislations, which became known as Basel Accords I and II, connecting VaR with the minimal capital requirements for commercial banks, set aside to protect against market risk.

In less developed markets, the impact of banks regulation changes and VaR models, has not been studied good enough. Also, not all the member countries in EU 27 have conducted research what are impact and consequences because of changes in the banking sector. EU new member states lag behind the developed EU markets in many fields, particularly in subjects like market discipline, financial legislation, disclosure of financial and other information, insider trading, embezzlement, knowledge of markets, financial instruments and risks. Market regulators, investment funds and banks use the same models for measuring market risk and capital reserves requirements in both developed and new members markets, supposing equal behaviour and characteristics. Serious doubts can arise when the results of these VaR measures take leading role for risk governance and management. New EU member states have generally higher volatility and lower liquidity. Most of the different VaR calculation methods assume normal market, normal distribution of returns, no trading during the holding period and
liquidity of the assets (Jorion 2000). In the present turbulent economic reality these preconditions does not hold and it is important to check how VaR measures behave in extreme volatile environment, if the mechanism of risk prediction have to be improved.

In the new EU member states, the alternatives to standard banking loans like issuing shares (preferential and ordinary) and debt securities (bonds and commercial papers) is increasingly popular. Because of possibility to earn high profits, the new tempting alternatives are attractive for all kind of foreign investors too (for companies, households, pension funds and banks). However, investors in these rapidly growing markets are not aware of potential important risks they can meet. It is easier to understand when small investors underestimate these risks, but then it comes to big institutional investors, the situation when they invest in transitional equity markets and choose wrong measuring instruments is not understandable and troubling. (Zikovic 2007:326).

1.1 Purpose of the study

The purpose of this thesis is to compare commonly used VaR measures calculated through Historical and Monte Carlo Simulations and to answer the question whether these VaR measures adequately capture market risk in EU new member country. It will also compare VaR behaviour during the global recession with previous terms of high volatile markets as well as stable growth periods with relative calmness of trade. The behaviour of these various models is analyzed over the simulation period with the help of a range of backtesting procedures to examine how accurately the models meet the specified confidence intervals. Backtesting is applied for ten years of historic price data of five major world market indices and the Bulgarian stock exchange index SOFIX in order to assess the accuracy of VaR predicting ability.

This study also focuses on contributing to the ongoing discussion about the usefulness of single risk measure both for traders and regulatory bodies as more and more governments are looking toward reintroducing tighter monitoring and control over financial conglomerates.
The study questions whether VaR measures underestimate the existing risk during periods of market turbulence and high volatility and overestimates risk during periods of low volatility showing inefficiency to measure exceptional market movements and how effective they are in capturing market risk in EU new member state. Based on these questions the following hypotheses of this study are set:

Hypothesis I Stock Index returns in new EU member state are normally distributed.

Hypothesis II Commonly used VaR measures adequately capture market risk in new EU member state.

Hypothesis III VaR underestimates risk during periods of high volatility and overestimates risk during periods of low volatility showing inefficiency in measuring extreme market movements.

1.2 Outline of the study

In order to examine the validity of these hypotheses the VaR calculations are done on the daily returns of the prices of five world major market indices S&P 500, FTSE 100, NASDAQ, Dow Jones, DAX, Stockholm General and compared with SOFIX, the market index of the new EU member Bulgaria. In chapter two, definitions of general types of financial risk are presented for detailed overview of the topic. The theoretical framework and the history of VaR together with follow up of the development of VaR as standard for financial regulation are presented in chapter three. It also lists different public and private organizations and institutions adopting VaR for their needs. The existing literature on the topic and previous empirical studies are examined in the following chapter four to build on discussion of historic evidence in favour and against this risk measure. Chapter five is devoted to the methodology on which the calculations in this paper are based and the different approaches to backtesting VaR effectiveness in covering risk. The used data is described in chapter six, and calculations together with the obtained results are presented in chapter seven. Historical VaR values are calculated
for years 2000-2010. Based on a moving window of 100 and 250 previous trading days data VaR estimation will be calculated and compared with the real market realization, counting the number of times VaR measure under-predict the daily loss. Additionally for each index two confidence levels of VaR, 95% and 99%, losses are compared. Chapter eight provides a number of concluding remarks and gives suggestion for future research.
2 GENERAL TYPES OF FINANCIAL RISK

The definitions for risk found in finance theory state it as the dispersion of unexpected outcomes due to movements in financial variables. It is measured by the standard deviation of unexpected outcomes, which is sigma (σ), or also called volatility. In financial markets volatility risk is the probability of fluctuations in the exchange rate of currencies. It actually is a probability measure of the threat that an exchange rate variation poses to an investor's portfolio in a foreign currency. Standard deviation over a dataset of exchange rate movements measures the volatility of the exchange rate. (Basel Committee on Banking Supervision, BSBC 2006.)

Choudhury (2003) defines risk as a probability that outcomes could be damaging or it can result in a loss. In the presence of risk, the outcomes can have some level of uncertainty. Horcher (2005:2) define risk as a probability of loss, which is a result of exposure. Any type of risk associated with financial operations can be classified as financial risk. It can arise when business transactions, such as sales, loans or investments are initiated. Financial risk is a result of sudden price changes, financial fluctuations, interest rate changes, internal and external organisational actions and failures. (Horcher 2005:2).

To distinguish between different causes of uncertainty Jorion (2000:14) describes five types of financial risk: market risk, credit risk, operational risk, liquidity risk and legal risk.

2.1 Market risk

Market risk appears when financial market prices and rates are changing. It could be absolute, measured in currency units, e.g. Euros, and relative, the risk which is measured in comparison with a benchmark index (Jorion 2000: 14). According to Crouhy, Galai & Mark (2006: 26) there are 4 types of market risk: interest rate risk, foreign exchange risk, equity price risk and commodity price risk. Interest rate risk is the risk that fixed-income security value could fall because of interest rate changes. This
risk can also appear when the yield curve shape is changing because of the long-term and short-term interest rates. (Horcher 2005:27) Foreign exchange risk is the risk of holding or taking positions in foreign currencies, including gold. This risk arises because of translation and transaction operations. Translation risk could appear when the financial statements of the balance sheet (e.g. assets, liabilities) are translated from the one currency to another. Transaction risk can appear from ordinary transactions operations which can impact the profitability of the company. (Horcher 2005:30). Foreign exchange risk appears from fluctuations in currency or international interest rates. The multinational corporations are mostly suffering from this kind of risk. It can result in huge losses in investments returns and in order to avoid it, the daily observations of the changing exchange rates are needed. (Crouhy 2006).

Equity price risk consists of two components: “general market risk” and “specific” of “idiosyncratic” risk. “General market risk” refers to a risk when changing broad stock market indices changes the financial instrument and portfolio value. “Specific” or “idiosyncratic” risk refers to volatility of the stock’s price that is determined by specific characteristics of the firm (breakdown in production process, quality in management, etc.). The difference between these risks is that portfolio diversification can eliminate general market risk, but cannot eliminate “specific” or “idiosyncratic” risk. (Crouhy 2006).

Another type of risk arising from holding or taking positions in commodities in the meaning of physical product traded on a secondary market, including precious metals, but excluding gold (treated as a foreign currency) is called commodity risk. Horcher (2005:35) divides commodity risk in two parts: commodity price risk and commodity quantity risk. Commodity price risk appears when the commodity prices are changing. This risk affects consumers and commodity producers. The rising commodity prices bring less profit for purchasers and the declining prices affect the revenues of producers. Commodity price risks in general have a high volatilities and large price discontinuities. (Crouhy 2006). The price risk in commodities is comparatively more sophisticated and volatile than that associated with currencies and interest rates. Commodity markets are not so liquid like those for interest rates and currencies. As a consequence, changes in
supply and demand can have a more striking effect on price and volatility. Effective hedging of commodities risk can be more demanding because of these market characteristics creating diminishing price transparency (BCBS 2006). This risk can be overcome, by offering commodity for domestic currency prices or allowing customers to calculate price by fixed exchange rate. It would be useful for smaller organizations, which cannot manage risk by themselves (Horcher 2005:36). Commodity quantity risk can appear when market demand is not equal to supply. The producers can get losses from the too high or too low demand (Horcher 2005:36).

2.2 Credit risk

Credit risk is the main risk of banks, because they are taking credit risk when lending money in exchange to some return. Banks have a greatest challenge to manage their credit risk (Lore & Borodovsky 2000). Credit risk is associated with the probability of default from one of the parties to perform its obligations. Credit risk can appear in several forms. One, called sovereign risk, specific for countries, it is the events occurring when government restrictions are imposed on currency. Other one, settlement risk can appear in transaction payment from both sides made in the same day when one side fails to deliver the payment (Jorion 2000:15). Variety of this risk is Counterparty Credit Risk. That is the probability that the counterparty to a transaction could default before it can deliver the final settlement of the transaction's cash flows. If the deal with counterparty has a positive value at the time of default then a loss would occur. In contrast with the unilateral exposure to credit risk through a loan, where only the lending bank faces the risk of loss, counterparty credit risk is a bilateral risk of loss. The market value of the transaction can have plus or minus sign to both counterparties. This market value can vary over time with the movement of underlying market factors (BCBS 2006).

A risk directly related to the counterparty credit risk is rollover risk. If transactions with a certain partner are expected to be carried on an ongoing basis in the future, but at the present moment they are not included in the calculations of positive exposure, the rollover risk arises. It can be two types: general, when the possibility of default on the
partner side is positively correlated to general market risk factors, or specific when the positive correlation is due to the nature of the transaction. An example of specific wrong-way rollover risk is when a future exposure to a specific counterparty is going to be high when the counterparty’s probability of default is also high. (BCBS 2006: 254-257.)

2.3 Operational risk

Operational risk appears from technical or human accidents or errors. It can be a result of management failure, wrong instructions or misleading information (Jorion 2000:17). Operational risk can be two types: operational strategic risk (external) and operational failure risk (internal). Operational strategic risk can appear because of not well chosen strategy in response to environmental factors, e.g. regulation, taxation, politics or competition. Operational failure risk arises from the failure of people, technologies or processes inside the company (Lore & Borodovsky 2000: 344). Operational risk could be also a model risk which appears when inappropriate model or inadequate framework is used or applied for the wrong purpose. (Jorion 1997:16).

2.4 Liquidity risk

Liquidity is a term for describing financial instruments, their markets and the companies’ financial abilities. It is the ability to make the payments and financial transactions. (Lore & Borodovsky 2000:443) Liquidity risk is a risk which arises when transaction cannot be done at dominant market prices, (Jorion 2000:17), when the ability to sell or purchase obligations or securities is limited (Horcher 2005:44), or when the company or financial institution runs out of liquid assets to finance the operations. The management of liquidity risk includes the understanding the place of liquidity of organisation, and how to deal liquidity, in the critical situations, e.g. when the shortfall arises. (Lore & Borodovsky 2000:443.)
In order to reduce liquidity risk, the company could reduce unusual or highly customised transactions, also transaction where liquidity depends on a small number of players. (Horcher 2005:44).

2.5 Legal risk

Potential loss arising from failure of a transaction due to improper legal or regulatory authorities’ proceedings qualifies as legal risk. This type of risk has direct links to credit risk. Market manipulations, insider trading and suitability restrictions are well known examples of such illegal activities. It can come from a foreign country with inadequate bankruptcy protection or from a court decision against a counterpart in transaction. (Jorion 2000: 18.)

2.6 Systemic risk

The risk that whole financial markets will fail to operate or will operate inefficiently is called systemic risk. Recent example of systemic risk was the collapse of Lehman Brothers that marked the peak of the recent economic recession. It had a domino effect on many banks and insurance institutions over the globe to spread uncertainty and volatility in gigantic proportions. The trust in all market mechanisms was undermined and entire financial system was in jeopardy. It is difficult to evaluate such risk because of the infrequent nature of such rare events. Derivatives allow for the spreading of risk across previously unrelated markets, raising the probability of large shock transition (with negative magnitude) from one market to others. Systemic risk of payment systems refers to the risk that liquidity problems at one financial institution will be passed on to others.

2.7 Other types of risk
Concentration risk is dealing with the dispersion of a bank's outstanding accounts over the variety of debtors to whom the bank has lent money. A "concentration ratio" is used to measure this risk. This ratio examines what percentage of the outstanding accounts each bank loan represents. When loans are moved toward a specific economic sector that would move the ratio higher than a portfolio of evenly spread loans, because greater diversification offsets the risk of economic downturn and default in any specific industry. An important factor in concentration risk is the risk of default. The most significant issue for the banks is whether bank's outstanding loans is equal to the overall risk posed by the economy as a whole, or are the bank's loans concentrated in areas of higher or lower than average risk. (BCBS 2006.)
3 THEORETICAL FRAMEWORK OF VAR

Holton (2003) made the first distinction between risk measures and risk metrics. Duration, delta, beta, and volatility are all risk metrics, and the procedure by which they are calculated is called risk measure. It is important to make such distinction because there is no exact correlation between risk measures and risk metrics. There are many metrics of risk—volatility, delta, gamma, duration, convexity, beta, etc. Measure that supports a risk metric is referred to as a risk measure. Risk measures are categorized depending on to the risk metrics they support. (Holton 2003.)

Beta is one of the risk metrics generally applied in the equity markets. It measures the systematic risk of a single instrument or an entire portfolio. The notion of beta was first used by Sharpe (1964) as part of his capital asset pricing model (CAPM) presented on figure 1.

![Figure 1. Capital asset pricing model](image)

Beta represents the sensitivity of an instrument or portfolio to extensive market movements. The stock market or the general market index is assigned a beta of 1. Then portfolios are compared to it. If portfolio has beta equal to 2 it is twice more riskier than the market. Oppositey if the value is under 1 it will be less exposed to price moves than the general market index. The formula for beta is:


\[ \beta = \frac{\text{cov}(Z_p, Z_m)}{\sigma_m^2} \]

where the dividend equals the covariance between portfolio return and the market return, and the divisor is the variance of the market's return or the volatility squared. Simple returns method is used for calculating both quantities. Beta is usually calculated from historical prices time series. There is a possibility to construct beta portfolios which are negative. Different approaches to do it can include shorting stocks, holding stocks (e.g. gold mining stocks) which have a tendency to move opposite to market or setting up appropriate options spreads. Beta can be used sometimes while measuring a market risk of portfolio. However, beta cannot capture specific risk and its value can be misleading. If the portfolio price volatility has low correlation with the market index this will result in low beta value, but still prices fluctuation can be significant. (Jorion 2000.)

Quite often the change in the value of the underlying asset is the main source of market risk for derivatives. There are two measures, delta and gamma, dealing with sensitivity to these changes. Delta \( \Delta \) is most probably the widest used concept in risk management. Delta shows how small changes in the price of the currency, commodity or underlying asset, denoted by \( I \) in equation 2, affects the theoretical price of portfolio or an instrument \( P(I) \):

\[ \Delta = \frac{\partial P(I)}{\partial I} \]

Sensitivity analysis is also closely related with Delta concept. This concept was developed for options, but it can also be applied to cash positions and other derivatives. (Linsmeier and Pearson 1996:25.)

The conventional definition of Gamma ( \( \Gamma \) ) comes straight from the calculation method. Gamma is the second partial derivative of delta with respect of the value of the underlying asset, currency, or commodity. If we denote the spot price of the underlying
asset with \( I \) and the option price as a function of \( I \) with \( P(I) \), then gamma of the option can be calculated by equation 3:

\[
(3) \quad \Gamma = \frac{\partial \Delta(I)}{\partial I} = \frac{\partial^2 P(I)}{\partial I^2}
\]

By measuring Delta changes (which can appear from changes in commodity, currency or underlying assets) Gamma complements Delta. (Linsmeier and Pearson1996:26-27.)

Delta and Gamma may be the most important measures for derivatives concerning market risk sensitivity, but derivatives are additionally exposed to implied volatilities, interest rates, and the passage of time. These market factors are covered by the Greek measures vega, rho, and theta consequently.

Much more complex extension of CAPM and measures described above is the Value at Risk method, which helps to determine the actual risk exposure of a portfolio to multiple risk factors. VaR methodology is similar to traditional market risk measurement methods in such way that VaR uses measurement of the dispersion of an asset's return during predefined time window around the asset's average return during that same time. The difference is that VaR expresses the downside of that dispersion, the loss in a single figure, while traditional methods apply statistical analysis to determine the standard deviation of those returns. (Van de Venter 2000:186.)

3.1 Definition of VaR

The formal definition of VaR given by Jorion (2000:88) states:

“VaR describes the quantile of the projected distribution of gains and losses over a target horizon. In general form VaR can be derived from the probability distribution of a future portfolio \( f(w) \). At a given confidence level \( c \) VaR represents the worst possible realization \( W^* \) such as the probability of exceeding this value is \( c \):
or such that the probability of a value lower than \( W^* \), \( p=P(w \leq W^*) \), is \( 1-c \):

\[
1 - c = \int_{-\infty}^{W^*} f(w)dw = P(w \leq W^*) = p
\]

The number \( W^* \) is called the quantile of the distribution, which is the cutoff value with a fixed probability of being exceeded.”

Advantages of VaR include the simplicity and elegance of its result—a single money figure and its possible implication to any kind of assets. The disadvantages are its ignorance of all risk above certain level and its vulnerability to fraud by traders. Calculation can be difficult when a portfolio consist of many different instruments.

The first company that introduced VaR measure was Bankers Trust. Their risk-adjusted return on capital system (RAROC) adjusts profits for capital at risk, defined as the amount of capital needed to cover 99 percent of the maximum expected loss over a year. 1-year horizon is used for all RAROC computations, no matter what the actual holding
period is, in order to allow better comparisons across different types of assets (Jorion 2000:77).

At the early 90’s Dennis Weatherstone, the chairman of JP Morgan established the classical framework of VaR in a “4:15 report” that he required from his employees. It contained measurement of the boundaries of all company risks on one page, available within 15 minutes of the market closure. In 1993 on an annual conference the bank made its risk model public and vastly disseminated VaR. They made it separate from the mother company, establishing a consulting firm RiskMetrics to improve the methodology. (Nocera 2009.)

3.2 Regulatory Capital Standards

The level of regulation of free market is a sensitive topic that has been driven back and forth when free market does not meet the general public expectations. In a case of extreme systemic risk bank deposits are destabilized. Deposit insurances alone offer not enough protection and government guarantees and bail outs have to be introduced. Guarantees are necessary to protect small depositors who cannot efficiently monitor their bank. Such monitoring is complex, expensive, and time consuming. For these reasons a unified standard across countries is needed. (Jorion 1997:42-44.)

3.2.1 Basel Accord I

After a series of discussions in 1988 the central bankers of the G10 member countries and Basel Committee on Banking Supervision (BCBS) brought out set of rules for the minimal capital requirements of banks. These rules define a common measure of solvency to cover the credit risk and arrange revealing of information about the debtors of banks. The Basel Accord requires capital charge no less than 8% of the total risk weighted assets of the bank (Jorion 2000:45). The regulatory capital itself consists of two components:
Tier 1 capital or “core” capital includes only permanent shareholders equity and disclosed reserves. Disclosed reserves also include general funds of the same quality. The general definition of capital excludes revaluation reserves and cumulative preference shares. General loan losses reserves represent capital that is available to the bank to meet losses, and when they occur they can not be charged directly to the fund but must be taken through the profit and loss account (BCBS 1995).

Tier 2 capital or "supplementary” capital includes perpetual securities, undisclosed reserves, subordinated debt with maturity longer than five years, general provisions or loan loss reserves held against future. Since long term debt has a junior status relative to depositors, debt is considered to be a buffer to protect depositors (BCBS 1995).

At least half of the total capital charge of 8% must be covered by tier 1 capital. The capital divided by the risk weighted assets is referred to as risk based capital ratio and upon it the following restrictions were set: 4% minimum on tier 1 capital and 8% minimum on total sum of tier 1 and tier 2 capitals. Under this regulations banks assets were separated into different categories with appropriately assigned risk weights to reflect associated underlying risk.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Asset Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>U.S. Treasury and obligations</td>
</tr>
<tr>
<td></td>
<td>Cash held</td>
</tr>
<tr>
<td></td>
<td>Gold</td>
</tr>
<tr>
<td>20%</td>
<td>Cash to be received</td>
</tr>
<tr>
<td></td>
<td>Claims on banks</td>
</tr>
<tr>
<td></td>
<td>U.S. government agency securities</td>
</tr>
<tr>
<td></td>
<td>Agency</td>
</tr>
<tr>
<td></td>
<td>Municipal general obligation bonds</td>
</tr>
<tr>
<td>50%</td>
<td>Municipal revenue bonds</td>
</tr>
<tr>
<td>100%</td>
<td>Corporate bonds</td>
</tr>
<tr>
<td></td>
<td>Less developed country debt</td>
</tr>
<tr>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td></td>
<td>Real estates</td>
</tr>
<tr>
<td></td>
<td>Plant and equipment</td>
</tr>
<tr>
<td></td>
<td>Mortgage strips and residuals</td>
</tr>
</tbody>
</table>
The Basel Accord I tried to set limits on “excessive risk takings” by demanding reports for all positions larger than 10% of bank capital and capping this “large risks” to max 25%. The sum of all “large risks” should not exceed 800% of capital (Jorion 2000:47). Basel Accord I passed legislation in G10 countries in year 1992 and currently more than 100 countries have affiliated its regulations (BCBS 1995).

The expansion of VaR popularity did not stop and in 1993 in a report the international body of economic and financial issues (G30) recommended its use as a consistent daily market risk measure (Global Derivatives Study Group 1993).

Significant shortfalls of Basel Accord I were revealed shortly after unveiling. Inability to account for the portfolio risk diversification, credit risk can be diversified across issuers, industries or geographical locations. The regulations do not associate the “netting” effect, when bank matches borrowers and lenders, and in such way decrease the counter default risk. The poor recognition of market risk such as interest rate risk in the regulations was also blamed. Pricing of assets was measured as book values which may vary significantly from market values and the accounting lag will cover additional exposures of the balance sheet. (Jorion 2000:47.)

The BCBS took into consideration these shortfalls and started moving the methodology toward measuring market risk with the VaR approach.

3.2.2 The Standardized Method

The Committee allowed bank to choose between two different methodologies when it comes to calculating capital requirements for credit risk. The primer choice is the Standardized Approach. Under it VaR was computed for portfolios exposed to interest rate risk, exchange rate risk, equity risk, and commodity risk with exact guidelines for each of them, issued by the commission. These instructions covered all fixed-rate and floating-rate debt securities, including non-convertible preference shares The sum of all this categories equals the total bank VaR. The method name comes from the risk
3.2.3 The Internal Model Approach

Many banks have chosen to implement the second alternative to measure market risk, internal model based approach. This model suggested the bank’s internal assessment of risk to serve as input for the capital reserves calculation. When introduced back in 1995 the internal model approach gave for first time to big banks the opportunity to use their own risk management systems to determine their capital charges. The computation of VaR under the Internal Model was based on a set of uniform quantitative inputs

- a horizon of 10 trading days, or two calendar weeks
- a 99 % confidence interval
- an observation period based on at least one year of historical data and updated at least once a quarter. (Jorion 1997:50.)

Internal model accounts for correlations in broad categories such as fixed incomes, as well as across categories between fixed income and currencies. The capital charge, according to the model, was set at the higher of the previous day VaR or the average VaR over the last 60 days times a “multiplicative” factor. The mathematical equation (6) representing the general market risk charge on any day \( t \) is:

\[
MRC_t = \text{Max}\left( k \frac{1}{60} \sum_{i=1}^{60} \text{VaR}_{t-i}, \text{VaR}_{t-1} \right)
\]

where \( k \) is the supervisory determined multiplicative factor. Local regulators got the authority to set the exact value of this factor, but it must be no less than of 3. This factor was supposed to provide further protection against market environments that are more volatile than historical data suggests. A penalty component could be added to this multiplicative factor if backtesting shows that the bank internal model forecasts risks in a wrong way. The possibility to change the value of this factor should stimulate banks
to improve the accuracy of their prediction models and to avoid optimistic estimations of profits and losses due to model fitting. The whole system was designed to reward truthful international monitoring and establish sound risk management systems. (Jorion 1997:50.)

\[
(7) \quad k = \begin{cases} 
3.0 & \text{if } N \leq 4 \quad \text{green} \\
3 + 0.2(N - 4) & \text{if } 5 \leq N \leq 9 \quad \text{yellow} \\
4.0 & \text{if } 10 < N \quad \text{red}
\end{cases}
\]

In order to gain total capital adequacy requirements, banks were required to sum up their credit risk charge together with their market risk charge, applied to normal operations. For the fact that additional capital have to be put aside, banks were allowed to use new class of capital, tier 3, which consists of short term subordinated debt. The amount of tier 3 capital was limited to 250% of tier 1 capital allocated to support market risks. (Jorion 1997:52.)

Internal Model Approach demanded from the national authorities to put substantial efforts to ensure consistency in its application. To qualify as eligible to use the Internal Model banks had to fulfil a list of quantitative and qualitative requirements. The Committee intentions to monitor and review the way the framework was introduced led to the establishment of Accord Implementation Group (AIG) for promoting framework’s application consistency. (BCBS 1996.)

The advantage of Internal Model Approach was pointed out to be the possibility to use regulatory capital requirements to supervise the moral hazard risks that appear when banks benefit from mispriced safety net guarantees or from the “too big to fail” notion.

3.2.4 Basel Accord II

The second issue of BCBS originally published June 2004 as Basel accord II revised the framework of recommendations on banking laws and regulations. It is applied to all internationally active banks ensuring that capital allocation is more risk sensitive. It
meant to preserve the integrity of capital in banks and assure that the risks of the whole banking group are captured. This framework differentiated operational from credit risk and quantified them both following VaR methodology. It tries to line up economic and regulatory capital closer to each other and to reduce the possibility for regulatory arbitrage. Also, it enables supervisors with standardized tools to test if individual banks are adequately capitalised on a stand-alone basis with the principal objectives of protection of depositors ensuring them that the capital recognised in capital adequacy measures are available. (BCBS 2006:2.)

The key elements used in the 1988 capital adequacy framework are still kept by the Committee. The requirements for banks total capital reserves have to be equal to minimum 8% of all assets which were risk-weighted. This represented eligible capital definition. However, the Committee wants to find better requirements which can take into account more risk-sensitive capital.

Basel Accord II uses a "three pillars" concept. The first one deals with the definition of eligible regulatory capital, minimum capital requirements addressing credit, operational and market risk. For computing each of them there is a choice of tools offered: credit risk can be measured using standardized approach, in a standardised manner, supported by external credit assessments. Foundation "Internal Rating-Based Approach" (IRB) which is subject to the explicit approval of the bank’s supervisor, would allow banks to use their internal rating systems for credit risk. Bank can also use advanced IRB. Similarly, there are three ways to calculate operational risk: basic indicator approach (BIA), standardized approach (TSA), and the internal measurement approach. For market risk the proposed approach is VaR. The second pillar treats supervisory review and introduces the key principles of risk management guidance and supervisory transparency. There are four of them;

"Principle 1: Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels.

Principle 2: Supervisors should review and evaluate banks’ internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their
compliance with regulatory capital ratios. Supervisors should take appropriate supervisory action if they are not satisfied with the result of this process.

Principle 3: Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum.

Principle 4: Supervisors should seek to intervene at an early stage to prevent capital from falling below the minimum levels required to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored.” (BSBC 2006.)

Pillar two includes guidance related to treatment of interest rate risk in the banking book, stress testing, definition of default, residual risk, and enhanced cross-border communication and cooperation. Supervisory standards allow supervisors to require buffer capital for risks, not covered under pillar one. (BCBS 2006:204.)

Pillar three reveals qualitative and quantitative disclosure requirements for the different model approaches choices in pillar two. It aims to improve market discipline by developing sets of disclosure requirements which allow market participants to assess relevant information on the scope of application, capital, risk exposures, risk assessment processes, therefore the whole capital adequacy of an institution. In such way comparison between banks is made easier. (BCBS 2006:226.)

The European Commission has adopted Europe wide capital requirements known as the Capital Adequacy Directive (CAD). The CAD was published 1993, laying down minimum levels of capital to be adopted for EU banks and security houses, extending the Basel guidelines. (CAD 2003.)

Indeed, Basel Accord II gave the biggest commercial banks more discretion in assessing capital requirements by reliance on their own internal methodologies and calculation results to set the capital requirements. This was a highly controversial deed in the light of the recent financial regulators malfunction.

3.2.5 Backtesting
Backtesting is a statistical procedure designed to compare realised trading results with model generated risk measures in order to evaluate the accuracy of the model (RiskMetrics 2000:39). It matches daily profits and losses with the VaR measure to assess the quality of banks risk modelling. Backtesting means counting the number of times that the trading return outcomes were larger than the risk measures. The actual covered fraction can then be compared with the intended level of coverage. If the comparison is close enough backtesting leads to no other issues, but if significant differences are revealed, immediate actions have to be taken. Banks can be asked to raise additional capital reserves and improve their risk management system. According to BCBS (1996:5) the backtesting framework should involve the use of risk measures calibrated to one-day holding period. It requires formal testing and accounting of exceptions on a quarterly basis using the most recent twelve months of data. The backtesting results are put into green, yellow or red zone according to the numbers of exceptions. This traffic light system signals the quality of banks risk models. Sorting results into zones take into account both the probability of rejecting an accurate model and accepting inaccurate model to be true. (BCBS 1996.)

### Table 3. Backtesting result zones (Source: BCBS 1996)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of exceptions</th>
<th>Increase in scaling factor</th>
<th>Cumulative probability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Green Zone</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8.11%</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>28.58%</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>54.32%</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>75.81%</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>89.22%</td>
</tr>
<tr>
<td><strong>Yellow Zone</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>95.88%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>98.63%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.65</td>
<td>99.60%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>99.89%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.85</td>
<td>99.97%</td>
<td></td>
</tr>
<tr>
<td><strong>Red Zone</strong></td>
<td>10 or more</td>
<td>1</td>
<td>99.99%</td>
</tr>
</tbody>
</table>

Using the number of exceptions as the main reference point in backtesting process is simple and straightforward approach that appeals to most supervisors and regulators. (BCBS 1996:14.)
Backtests are independent of VaR calculating process and can show whether VaR calculation methods exhibits correct unconditional and conditional coverage. This process is used as an indicator to the quality of the bank risk management and is referred as “backtesting”. (BCBS 1996:1). Institutions like The Global Association of Risk Professionals (GARP), General Accounting Office (GAO), The Financial Accounting Standards Board (FASB), Securities and Exchange Commission (SEC) found backtesting useful for developing their own risk measurements. Regulators could penalize banks by increase in the scaling factor when calculating their market risk provisions, due to the use of a risk measurement model with a lot of errors. (Zikovic 2007:326)

Several individual cases of market collapse of big hedge funds happened, like Long Term Capital Management “LTCM”, which looking retroactively to the recent financial crisis is similar and closely connected with taking excessive risk (Das 2008). Also Stock market crash of 1987, 1990’s “Junk bond crisis”, the Mexican, Asian and Russian market crises took the sophisticated risk models by surprise. Extreme events tend to happen much more often than the probabilities values that they were given, and though academic research pointed out most malfunctions of VaR measurements little was done for the practical implementations of these warnings, as Satyajit Das (2006) mentioned:

“A number (VaR) is produced for an audience that unquestioningly accepts it at face value and are content.”

The debate escalated recently to a level that certain theorist like Taleb (2007) advocate for complete abolishment of VaR as representative measure of risk, stressing that certainty and randomness applied to real world conditions may mislead us to a tragic proportions. Traders also started to agree with such extreme statements, which is prudent contemplating mind spinning write offs in the balance sheets. (Mihn 2008: 2)

“A 99% VaR calculation does not evaluate what happens in the last 1%. This, in my view, makes VaR relatively useless as a risk management tool and potentially catastrophic when its use creates a false sense of security among senior managers and watchdogs” (Einhorn 2008).
However, the scale of the crisis suggests there is not only one single factor, e.g. Value at Risk concept, that can be blamed for and it is more likely that the human greed and abuse of the regulations led to crash (Nocera 2009). Testing VaR models again with the market data from the biggest recession since WWII is interesting and important for evaluating the real reasons.
4 LITERATURE REVIEW

The theoretical framework for modern portfolio theory and foundations of efficient portfolios were established by Markowitz (1959). His mathematical modelling gave optimized solution for reducing risk by combining different financial instruments that are not perfectly positively correlated. Later, Sharpe (1963) introduced Asset pricing model, or the single index model as it came became known, to measure the risk and return of stocks by connecting their performance to market indices. These initial basic works and their modifications set the foundations for the introduction of VaR and advanced risk management techniques. The interest toward developing risk management in the early 90’s, after the stock market crashes, was boosted by regulatory authorities who set risk-based capital adequacy requirements on financial institutions (cf. Dimson & Marsh 1995). The improvement in knowledge about risk management was marked by few significant research work and studies. In academics, Engle (1982) pioneered the development of volatility models for measuring and forecasting volatility dynamics. In business world the introduction of RiskMetrics by J.P. Morgan (1996) standardized the measures used by traders to compute credit and market risk (viz. RiskMetrics Tehnical Document 1996). As Riskmetrics offered a benchmark methodology, the interest of academics was set upon testing alternatives and making possible improvements. Still most of the early analyses like Beder (1995), Hendricks (1996), Marshall and Seigel (1997) and Pritsker (1997) were limited to examining the difference between modelling approaches and explaining implementation procedures with the use of illustrative portfolios. Notable research work was conducted by Christoffersen, Harm and Inoue (2001). They developed a testing procedure for the acceptability of VaR method measuring the confidence level with the actual realized percentage of VaR breaks realized loss bigger then VaR prediction and described how VaR measures can be compared. The authors used GARCH and Riskmetricks models based on daily returns for S&P 500.

Different statistical methodologies for evaluating the accuracy of VaR models emerged, namely evaluation constructed on the binomial distribution of VaR breaks proposed by
Kupiec (1995), distribution forecast evaluation as introduced by Crnkovic and Drachman (1995) and Christoffersen (1998) interval forecast evaluation. Lopez (1995) proposed methodology, based on probability forecasting framework. A new evaluation discussed by, is proposed. This methodology gauges the accuracy of VaR models using forecast evaluation techniques. It provided regulatory agencies more flexibility to defining appropriate loss function. These methodologies count for whether the VaR forecasts display properties of accurate VaR forecasts.

Campbell (2005) examined variety of backtests which analyzed how adequate are VaR measures. Backtesting was reviewed from different angles, from the risk management and statistical contexts in his paper. Backtests were sorted in three groups: in one group, the ones which examine the independence property of VaR measure, other group examining the unconditional coverage property and the last group contains backests which examine both properties. Statistical power properties of these tests were analyzed in a simulation experiment. Ultimately, backtests shortfalls were discussed. (Campbell 2005.)

Duffie and Pan (1997) concentrate their attention on market risk associated with changes in prices or rates of underlying trade instruments over short time horizons. Whether VaR is accurate measure of the risk of financial distress over a short time period depends on the liquidity of the portfolio of positions and global market liquidity in general. Most common VaR models suggest normal distribution or of price changes, but the studied data for commodity and equity markets, exchange and interest rates revealed significant amounts of positive kurtosis. The authors concluded that the probability distributions of daily changes in these variables have “fat tails” and so extreme outcomes appear more often than the normal distribution assumption anticipates. Duffie and Pan pinpointed jumps and stochastic volatility as probable causes of kurtosis. They showed that under a jump diffusion model kurtosis is a declining function of the time horizon. If a stochastic volatility model is used, kurtosis is an increasing function of the time horizon normally used in VaR calculations. (Duffie and Pan 1997.)
The issue of relaxing normal distribution assumption was also addressed by Hull and White (1998). In the model they proposed, users are allowed to choose any probability distribution for the daily changes in each market variable. The parameters of the distribution were subjected to updating schemes such as GARCH. Distinctive feature of this model is the way it treats correlations. Daily changes in each market variable were transformed into a new variable that is normally distributed. After this conversion the new variables are considered multivariate normal. Tests of the model were done with data for twelve different exchange rates over nine years of price history, and the authors reported that the predictions for distributions of daily changes based on the first half of the data window were already good enough for the second half of the period. (Hull and White 1998.)

After BCBS gave the large banks privilege to use their own internal risk models to set up capital reserves with the second Basel Accord, a major challenge for banks regulators became the verification of the accuracy of this models. Verification is viewed as a protection that banks will not try to speculate with the regulation rules by model fitting. Berkowitz and O’Brien (2002) were one of the first to evaluate the actual performance of the banks trading risk models by examining the statistical accuracy of the VaR forecasts. They used data of 6 large U.S. banks that followed Basel regulations and did calculation for a 99 percent lower critical value of aggregate trading profit and loss (P&L) with a one-day horizon. They evaluated the VaR forecasts by testing null hypothesis of a 99 percent coverage rate and discovered that VaR estimates tend to be conservative relative to this percentile of P&L. Banks showed tendency to overstate VaR figures in fear of higher capital reserves sanctions. Berkowitz and O’Brien reported that tested VaR models do not include forecasts of net fee income, but this fee is included in daily P&L, and this practice gives the VaR forecasts a conservative bias. Nevertheless, authors also observed that at some times losses can largely exceed VaR and such extreme events tend to be clustered. The authors concluded that banks models, with all their approximations for the thousands of existing risk factors, have troubles predicting changes in the volatility of P&L.
Berkowitz and O’Brien (2002) results suggest that the bank VaR models do not contribute to a better forecasts more than simple GARCH model of P&L volatility that grants comparable risk coverage with less regulatory capital. This is due to the GARCH model higher responsiveness to changes in P&L volatility. P&L correlations across banks are potential concern to bank regulators because of the thread of systemic risk, the simultaneous realization of large losses at several banks. For evaluating exposures to liquidity or other market crisis, banks are limited to stress testing. Reported VaR conservative estimates suggest greater levels of capital coverage for trading risk, but they are less useful as a measure of actual portfolio risk. (Berkowitz and O’Brien 2002.)

To find answer to the question which method of calculating VaR works best, Linsmeier and Pearson (1996) compared Historical simulation, the variance-covariance method and Monte Carlo simulation on the following list of criteria: ability to capture the risks of options and option-like instruments, ease of implementation, ease of explanation to senior management, flexibility in analyzing the effect of changes in the assumptions, and reliability of the results. For each approach they briefly described the procedure how to perform the analysis on single instrument portfolio and how it can be expanded for multiple instrument portfolio. Linsmeier and Pearson identified great variations across methods and could not provide clear answers. The two simulations they tested, Historical and Monte Carlo Simulation, seemed to work well for combined portfolios of options because they recalculated the value of the portfolio for each "draw" of the basic market factors and have good estimation for the statistical distribution, though, wrong choices could lead to potential error in Monte Carlo simulation and the obtained VaR results. Similarly, the distribution of Historical Simulation would be misleading if the historical samples were not enough representative.

Linsmeier and Pearson reported that atypical price period (e.g. low volatility) produced lower value VaR and the authors explained how these biases can be exploited by informed traders to take more risky positions than the regulators have intended. The study displays that historic data contains unreasonable approximation of future. Authors concluded that in case one wants to incorporate what-if scenarios, MCS is most appropriate choice. Linsmeier and Pearson (1996) explained the theoretical possibility
to include in both simulations randomness of volatility though they did not actually test it. HS was found to be the easiest method to implement and explain to senior managers and shareholders, while MS was hardest. VaR methodology was found to be not useful for smaller nonfinancial corporations that might find it difficult to combine such complex measure. Instead "cash flow at risk" or simple sensitivity analyses were proposed to be adequate for SME’s. (Linsmeier & Pearson 1996.)

More recent comparison between VaR methods was done by Yamai and Yoshiha (2002). Focusing on tail risk, they illustrate how it can bring serious practical problems. They found out that information given by VaR may mislead rational investors, and employing only VaR as a risk measure is likely to result in a larger loss in the states beyond the VaR level. Their research works show that if the event window chosen for the simulation does not contain enough volatility, then accordingly VaR number will be smaller, so longer time horizon would be preferable. Furthermore, the usability of VaR for longer time periods was studied by Culp, Mensink, and Neves (1998) who found that it gives practical information for estimation of market risk for multicurrency asset managers. Comparing 100 days with 250 days of trading windows, Beder (1996:12) also found that the shorter one appears to be inadequate. Nevertheless, these results showed significant bias due to the fact that he used small data sample and this made the left tail of the distribution tricky to measure. Longer data windows usage was encouraged by the empirical research of Hendricks (1996), in which the longest samples produced the best performance. However, there is problem with longer data periods, precisely the fact that even one unusual return keeps volatility estimates high, and this abnormal high estimation will last the same amount of time afterwards, even though the underlying volatility may have returned to normal levels, as was shown by Alexander & Leigh (1997:53).

Pritsker (2006) inspected the HS method and the Filtered Historical Simulation (FHS) method. With changes in conditional risk both methods were not reacting fast enough. It turned out also, that while large losses increase VaR measure, large gains (suggesting highly risky positions) do not affect it at all. Pritsker (2006) comparison showed FHS method to be better though its risk estimates were varying in small samples. The
assumption of constant correlation in FHS methodology was proven to be false for large samples, and the author argues that both methods needed additional refinements for time-varying correlations. This problem is directly related to the choice of appropriate length of the historical sample period. (Pritsker 2006.)

Basak and Shapiro (2001) analyzed optimal, dynamic portfolios exposed to market-risk using VaR. They reported that VaR risk managers often optimally choose a larger exposure to risky assets than non-risk managers and consequently incurred larger losses when losses occur. A general-equilibrium analysis revealed that the presence of VaR, risk managers amplifies the stock-market volatility at times of down markets and attenuates the volatility at times of up markets. An identified general shortcoming of VaR reveals to be the fact that it tells nothing about the size of loss when breaks occur. In this area Naryan (2004) also reported results showing that a large number of equity-oriented hedge fund strategies exhibit payoffs resembling a short position in a put option on the market index and therefore bear significant left-tail risk, risk that is ignored by the commonly used mean-variance framework. Using a mean-conditional value-at-risk framework, they demonstrate the extent to which the mean-variance framework underestimates the tail risk. (Basak and Shapiro 2001.)

Analysing Basel II accord, McAleer (2008) found evidence that it encourages risk taking at the expense of providing accurate measures and forecasts of risk. The author suggests the improvements of optimal strategies in a risk monitoring and management. (McAleer 2008.)

Studying VaR disclosure, Perignon and Smith (2010) formed an index from six components of VaR: holding period and confidence level, summary statistic, previous years summery, definition of trading revenue and backtesting. For the calculation of index value they used data for ten US & Canada banks, covering a period of ten years. The authors found out that Historical Simulation proofed to be by far the most popular method for computation globally with most of the banks official report using it. Pie chart in figure 3 displays the relative frequency of each VaR calculation method used by sample banks.
They also detected persistent overstatement of VaR, which lead to too few exceptions. This fact is explained to have a direct connection with the Basel Accords minimal capital requirements capital. The comparison showed that the quality of the revealed VaR information indicates little to no improvement over time. Authors conclude that VaR result from historical simulation does not contain a lot useful information about the future volatility. (Perignon & Smith 2010.)

The impact of Basel Accords allowing banks to calculate their capital requirement based on their internal VaR models and the consequences of regulation changes on banks in New EU member states has not been studied well enough. Limited number of papers tested VaR models in developing stock markets. Significant examples are Santos (2000) for Mexico stock exchange, Sinha & Chamu (2000) for Indonesia, Fallon & Sabogal 2004 for Columbia, and Valentinyi-Endrész (2004) used Hungarian market data. Zikovic (2007) compared the performance of historical simulation VaR models on stock indexes of some new EU member states and candidate countries indices, CROBEX (Croatia), SOFIX (Bulgaria), BBETINRM (Romania) and XU100 (Turkey). Backtesting results show that VaR models adopted from developed stock markets are not particularly accurate for measuring market risk in the studied countries.
Gencay and Selcuk (2004) collected daily stock market data from Argentina, Brazil, Hong Kong, Indonesia, Korea, Mexico, Singapore, Taiwan and Turkey to investigate the relative performance of Value-at-Risk (VaR) models with the daily stock market returns. In addition to the Variance Covariance method and Historical Simulation VaR, they also studied the extreme value theory to generate VaR estimates. Their results indicate that extreme value theory based VaR estimates have greater accuracy at higher quintiles.
5 METHODOLOGY

The two simulations that will be conducted in this study, Historical and Monte Carlo, belong to the full valuation VaR methods. They measure risk by changing the full price of a portfolio over different scenarios. The historical simulation method consists of going back in time and applying current weights to a time-series of historical asset returns. This approach is also known as bootstrapping because it involves using the actual distribution of recent historical data (Jorion 2000). Let \( r_t = \log\left(\frac{p_t}{p_{t-1}}\right) \) to be the returns at time \( t \) where \( p_t \) is the price of an asset or a portfolio at time \( t \). VaR, \( (\alpha) \) at the \((1 - \alpha)\) percentile is defined by:

\[
(8) \quad \text{Pr}(r_t \leq \text{VaR}_t(\alpha)) = \alpha
\]

which calculates the probability of returns at time \( t \) to be less or equal to \( \text{VaR}_t(\alpha) \), \( \alpha \) percent of the time.

5.1 Historical Simulation: Model description

Historical Simulation (HS) is an approach that does not need many assumptions about the statistical distribution of the underlying market factors (Linsmaier & Pearson 1996:7). As RiskMetrics describe it, it is as non-parametric method of using past data to make interpretations about the future. Applying HS technique means to take today’s portfolio or assets and re-value them using the past prices. In short, HS basis consists of taking actual historical price changes in market rates that occurred over the last trading days (that would be the data window) and re-values the asset or portfolio as if those changes were to occur again in the next holding period.

\[
(9) \quad R_{p,\tau} = \sum_{i=1}^{N} W_i R_{i,\tau}, \tau = 1, ..., T
\]
HS is a direct implementation of full valuation methodology where $R$ is historic asset return, $N$ is the number of portfolio position, $\tau$ observations times and $w$ - the weight of the position (Jorion 1997:193). Distinctively, a historical VaR value is calculated by using historical changes in market prices to construct a distribution of potential future portfolio profit and losses, and then reading off it the VaR number as the loss that is exceeded only a certain percentage of the time (Linsmeier & Pearson 1996:7).

HS approximates the quantiles of an underlying distribution from the realization of the distribution. The VaR in this case is estimated by:

\[ \text{VaR}_t(\alpha) = F^{-1}(\alpha) r \]

where $F^{-1}(\alpha) r$ is the $q$ th quantile ($q = 1 - \alpha$) of the sample distribution. Statistical calculations are simplified as this methodology uses the actual observed changes to estimate expected future market changes. A lot of financial models consider markets and prices of instruments to be continuous in nature and that there are no sharp jumps or discontinuities in prices. Since HS exploits actual returns, the method captures true market behaviour and does not rely on the assumption of normal distribution of market returns (Venter 2000:6). Changes in market prices are used as input to calculate prospective gains and losses, so any "fat-tails" or other distortions are fully captured in the model (Stambaugh 1996:617).
There are five key steps in conducting a HS for a single financial instrument: First the basic market factors that affect the instrument have to be identified. Second, formula containing these factors and expressing the mark–to–market value of the instrument in the portfolio has to be obtained. The next step is to extract the historical values of the market factors for the last period of interest (100 and 250 days). Subjecting the chosen instrument to the daily changes in market rates and prices of the factors create hypothetical profits and losses. Then a histogram of all profits and losses is created, and from it the desired percentile (1% and 5%) is subtracted (Linsmeier & Pearson 1996:15).

5.1.1 Advantages

HS method is relatively simple to implement if historical data is available for all the financial instruments over the time horizon for daily marking-to-market. Its intuitiveness makes it easy to explain to managers, supervisors and regulators. This assists in the process of acquiring conclusions from VaR analysis and supports the disciplines of putting risk management to work (Stambaugh 1996:617). As HS VaR is derived from actual prices, the method allows nonlinearities and nonnormal
distributions. It captures gamma, vega risk, and correlations. HS does not depend on specific assumptions about valuation models or the structure of the market and is not subject to model risk. Its biggest advantage is that it can account for fat tails (Jorion 1997:195). HS is also able to incorporate changes in option prices with changes in option volatilities if they are collected and included as additional factors for the period used in the simulation (Linsmeier and Pearson 1996:17). HS is the most widely used method to compute VAR.

5.1.2 Problems

HS does not go without shortfalls. It cannot be done without sufficient recorded history of price changes for all assets. Based on the data only one sample path is generated, and the quality of the results critically depends on the length of the chosen period. HS suggests that the past closely represents the immediate future. If the chosen time window misses important events, the tails will not be well represented or the period may contain events that will not reappear in the future. Linsmeier and Pearson (1996:19) refer to this problem as “atypical” historic data. One hidden danger of this fact is that if HS VaR is used as trading desk limit, this shortfall opens the door for informed traders to exploit the model and take additional risk. It is also hard to perform “what if” scenarios analysis when the price path is directly linked to historic changes. HS method puts equal weights on all observations in the window, without considering the influence that most recent ones have on the immediate future. Thus, HS will be very slow to incorporate structural breaks. (Jorion 1997:196.)

5.2 Monte Carlo Simulation: Model description

Monte Carlo method (MCS) uses simulation techniques to produce huge number of random price paths in order to estimate the behaviour of future assets prices. In such fashion it generates diverse scenarios for the value of a portfolio. MCS includes wide range of hypothetical values of financial variables and can also implement possible correlations between them (Jorion 1997:231).
Conducting MCS for a single instrument starts the same like HS with identification of basic market factors and formulating its value accordingly. Afterwards, a specific distribution for the changes in these factors has to be estimated by the modeller. One has freedom to choose the distribution parameters which fits best the objectives of the simulation. Consequently using the random number generator, one can create sufficient number (N) of hypothetical values of changes in the market factors. They are used to calculate N hypothetical mark-to-market values of the instrument. Subtracting the actual instrument value gives us N hypothetical profits and losses, and then again histogram is created and VaR value is taken from it (Linsmeier and Pearson 1996:15).

**Figure 5.** MCS Process flow chart. (Source: Jorion 2000)

HS and MCS are quite similar, except that in MCS the hypothetical changes in price are created by random draws from pre-specified stochastic process instead of sampled from real data set.

5.2.1 Advantages
MCS is one of the most powerful VaR methods. If modelling of parameters is accurate, MCS is really extensive and limited only by computational power. It is flexible enough to include time variation in volatility, fat tails, and extreme scenarios. It can also implement additional types of risk like price risk, volatility risk and to some extend model risk. MCS method can include also user-defined scenarios, nonlinear positions, nonnormal distributions and implied parameters (Jorion 1997:200.) Generally MCS is able to integrate price volatility randomness by extending the simulation in a manner that it includes a distribution of volatilities. All kind of “What-if” scenarios are compatible with MCS method. (Linsmeier and Pearson 1996:17.)

5.2.2 Problems

MCS is heavily influenced by model risk. It relies on stochastic processes for defying the risk factors and pricing models that can be arguable or totally wrong. Incorrect assumptions about the parameters of statistical distributions of market factors lead to mistakes in VaR values. A great deal of expertise and professionalism is needed to make the right choice. A portfolio with exotic options may be difficult to value if one lacks adequate pricing models. As finite numbers of price path replications are done, MCS VAR estimates can vary significantly from sample to sample. Another drawback is the fact that it is hard to explain it to senior managers. (Linsmeier and Pearson 1996:17.)
Table 4. Comparison of VaR methodologies. (Source: Linsmeier & Pearson)

<table>
<thead>
<tr>
<th>Comparison criteria</th>
<th>Historical Simulation</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to capture the risks of portfolios which include options?</td>
<td>Yes, regardless of the options content of the portfolio</td>
<td>Yes</td>
</tr>
<tr>
<td>Easy to implement?</td>
<td>Yes, for portfolios for which data on the past values of the market factors are available</td>
<td>Yes, for portfolios restricted to instruments and currencies covered by available &quot;off-the-shelf software. Otherwise moderately to extremely difficult to implement</td>
</tr>
<tr>
<td>Computations performed quickly?</td>
<td>Yes</td>
<td>No, except for relatively small portfolios</td>
</tr>
<tr>
<td>Easy to explain to senior management?</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Produces misleading value at risk estimates when recent past is atypical?</td>
<td>Yes</td>
<td>Yes, except that alternatives estimates of parameters may be used</td>
</tr>
<tr>
<td>Easy to perform “what if” analysis to examine effect of alternative assumptions?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

5.3 Value-at-Risk inputs

Calculation of VaR requires the input of the following variables:

5.3.1 Holding period

The time window for which eventual losses will be projected can be referred as VaR holding period. VaR as a measure is time specific and can indicate both long term, month or quarter portfolio risk, as well as overnight positions risk (Verder 2000:3). General rule is that longer holding period carries greater risk due to the fact that absolute volatility increases over time. Trading VaR, revealed by most of the commercial banks, uses one day horizon. One reason for this is the liquidity and fast
turnover in their portfolios, but most important, daily VAR allows to be easily compared with daily profit and loss (P&L) measures (Jorion 1997). As Minnich (1998:42) puts it VaR holding period should correspond to the time required to hedge the market risk. BCBS in Basel II proposed to regulators to scale from one day holding period to 10 days using the square root of time or $\sqrt{10} = 3.16$.

5.3.2 Confidence level

Confidence level represents the tolerance level for which the loss estimated by VaR value can be surpassed. The Bank for International Settlement recommends 99th percentile, one-tailed confidence interval to be used, but a lot of risk managers prefer to calculate VaR values over a 95% confidence level, such as used in the JP Morgan’s RiskMetrics methodology (Duffie and Pan 1997:9). The actual cost of a loss exceeding VAR and the degree of risk aversion are the main criteria when choosing confidence level. The bigger they are, the larger the need for capital reserves to cover possible losses. In such a case higher confidence level should be implied (Jorion 1997; Verder 2000). The choice of the confidence interval may not be so important if VaR is used to compare risk across markets. Researchers provide evidence that 95% confidence interval performs best under backtesting because of the existence of "fat-tails" (Minnich 1998:42).

5.3.3 Data window

When calculating VaR risk professionals have to choose how much historical data to include in the model, or how long the data window should be. Minnich (1998:43) argues that longer periods have return distribution containing more examples of extreme events, while shorter periods allow VaR values to react quicker to changing market.

5.4 Backtesting Value-at-Risk
The most common tests used nowadays to evaluate VaR measures of banks are the binomial method and the interval forecast method developed by Christoffersen (1998). Both of these tests utilize null hypothesis that VaR estimates display a specified property that characterize the estimate to be a correct one. If the null hypothesis happen to be rejected, that signs the VaR estimate is considered to be not accurate. If the null hypothesis is not rejected, then the particular VaR model is believed to be “acceptably accurate”. (Lopez 1999b:4).

As both tests focus on the comparison between the reported VaR estimate and the realized profit and loss over fixed interval of time they use a defined with mathematical expression as follows in equation 9 “hit” function;

\[
I_{t+1}(\alpha) = \begin{cases} 
1 & \text{if } x_{t+1} \leq -VaR_{t+1}(\alpha) \\
0 & \text{if } x_{t+1} > -VaR_{t+1}(\alpha) 
\end{cases}
\]

This function sequence denotes the history whether or not realizations of losses which exceed VaR had happened. The task of evaluating the accuracy of a given VaR model can be decomposed to determining if the hit sequence \([ I_{t+1}(\alpha) ]\) satisfy two properties, unconditional coverage property and independence property for the whole time interval \((t = 1; T)\) (Campbell 2005:3). The first property consists of the fact that the probability of loss realization must be exactly equal to \(\alpha \times 100\%\). If the percentage of losses surpass the VaR measure, this would imply that the actual level of risk is consistently underestimated. On the other hand, if there are hardly any violations of VaR, it might be a sigh of over conservative VaR measure. The Independence property sets restrictions on the frequency with which VaR violations occur. It imposes that any two elements of the hit sequence should be independent of each other. This rule means that the previous VaR violations should contain no additional information about the occurrence of new violation. In case those former breaches of VaR carry prediction about the possibility of future breaches happening, this points out to a major inadequacy of the VaR measure. An example of breaking independence property is the “clustering of violations” phenomena. (Campbell 2005:3.) Unconditional coverage and independence properties are separate of each other and correct VaR model have to satisfy them both. That
results in a combined statement for the “hit” sequence which reads that it has to be “identically and independently distributed (iid) as a Bernoulli random variable with probability $\alpha$ “.

$$I_{j}(\alpha) \approx B(\alpha)$$

(Campbell 2005:4)

5.4.1 Unconditional coverage test

One of the earliest unconditional coverage backtests, proposed by Kupiec (1995), is known as Proportion of Failure (POF) test. It became a standard way of backtesting different models used to forecast VaR. By counting the number of tail losses that exceeds VaR and comparing with the expected level of confidence POF test can statistically reject or accept the correctness of a particular VaR model. Theoretically, the setup tests whether the observed failure rate $\frac{N}{T}$ is sufficiently close to the left tail probability $P$ for the test not to reject the model. (Lopez 1999b.)

In this study the notations for this test are set as follows: observed number of tail losses exceeding VaR is denoted by $N$, the sample size is denoted by $T$. Since the probability of detecting $N$ exceptions in a sample sized $T$ is:

$$P(N) = \binom{T}{N} \alpha^N (1 - \alpha)^{T-N}$$

Thus, the relevant null and alternative hypotheses are:

$$H_0: \frac{N}{T} = P$$
H1: $\frac{N}{T} \neq P$

The applicable likelihood ratio $LR_{uc}$ statistical test as takes the form in equation 9:

\[
(14) \quad LR_{uc} = 2 \left[ \log\left(\frac{N}{T}^N \left(1 - \frac{N}{T}\right)^{T-N}\right) - \log\left(p^N \left(1 - p\right)^{T-N}\right) \right]
\]

$LR_{uc}$ test proved to have the greatest power for given sample size and under the null correct hypothesis of correct unconditional coverage the test-statistic has asymptotic $\chi^2(1)$-distribution with one degree of freedom. A model fails the test if the p-value is less than 0.05 using the standard significance level. (Lopez 1999b:6.)

A connection exists between the test of unconditional coverage and the capital requirements framework. The market risk capital multiplier is determined by the number of VaR violations in a historic period. In the same way the POF test is a function of the amount of VaR violations in the same period. As a consequence, there is a tight bound between the raising of the market risk multiplier and POF test. The role of the market risk multiplier is actually a test for unconditional coverage that continuously rise the capital reserves of banks with poor model predictions. (Campbell 2005:6.)

Apart from Kupiec’s POF test there exist simple statistical tests that take into account the unconditional coverage property of VaR models. A test can be based directly on the average value of VaR for given interval of time. Assuming that VaR is accurate then the scaled version of the time period has approximately standard normal distribution. The exact distribution is known to have value of $z$ (equation 10)

\[
(15) \quad z = \frac{\sqrt{T}(\hat{\alpha} - \alpha)}{\sqrt{\alpha(1-\alpha)} }
\]

and the same hypothesis test as for POF can be performed. (Campbell 2005:7.)
Tests for unconditional coverage are good benchmarks for evaluating VaR measures but have two major shortfalls. They have troubles detecting measures that systemically under report risk and do not examine for the independence property which, as underlined by Campbell (1995), can result into a significant risk or leverage effects and possible asymmetrical, clustered losses. That’s why the return sets have to be checked whether VaR breaks are spread across the sample and do not appear in clusters.

5.4.2 Conditional coverage test

VaR Estimates can be seen as interval forecasts of the of the left tail percentile of one step ahead return distribution. This interval forecasts can be inspected for both unconditional and conditional coverage. As was discussed earlier, $LR_{uc}$ test is unconditional. However in the presence of time varying heteroskedasticity in the returns, the conditional accuracy of interval forecasts is also of great importance (Lopez 1999b:6). Interval forecasts that overlook this variance dynamics may pass the unconditional test but will exhibit faulty conditional coverage. VaR model that lack the ability to take into account possible clustering of volatility are also likely to have a lot of breaks during intervals of high market turbulence. Inadequate volatility modelling leads to serial correlation in the breaks. (Lopez 1999b.)

Christoffersen (1998) proposed a testing framework that is autonomous of the process of generating the VaR estimates. It takes into consideration whether the VaR model shows correct conditional coverage. Christoffersen technique includes the following procedures for the evaluation of interval forecasts: test for correct unconditional coverage, test for independence and test for correct conditional coverage. The early test that account for the independence property of the “hit” function was based on the Markov chain process, which has the property that one state of such discrete random process depends only on the previous state. Accordingly the test examines whether or not the probability of VaR break on the following day depends from the appearance of break today. If VaR measure is correct then the probability of VaR violation occurrence now should be independent from whether or not break happened yesterday (Campbell 2005:8).
If a VaR model truly captures the conditional distribution of the returns, then breaks should be independently distributed over time. Tested against alternative of first order Markov dependence, the likelihood ratio will take the form:

\[ L_A = (1 - \pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1 - \pi_{11})^{T_{10}} \pi_{11}^{T_{11}} \]

\[ T_{ij} \] denotes the number of observations in state \( j \) after having been in state \( i \) the period before \( \pi_{01} = \frac{T_{01}}{T_{00} + T_{01}} \) and \( \pi_{11} = \frac{T_{11}}{T_{10} + T_{11}} \). Under the null hypothesis of serial independence

\[ H_0: \pi_{01} = \pi_{11} = \pi \]

the relevant likelihood function is

\[ L_0 = (1 - \pi)^{T_{00} + T_{01}} \pi_{01}^{T_{01}} \pi_{11}^{T_{11}} \]

Where \( \pi = \frac{T_{01} + T_{11}}{T} \), \( T \) is the total number of observations. The test statistic for independence has the form as in equation 14, and asymptotic \( \chi^2(1) \) distribution. (Lopez 1996b.)

\[ LR_{ind} = 2(\log L_A - \log L_o) \]

The test for correct conditional coverage \( LR_{cc} \) actually consists of two tests, one for correct unconditional coverage and one for serial independence. The necessary and sufficient condition for VaR model to pass it is they should be both satisfied. The relevant test statistic is:

\[ LR_{cc} = LR_{uc} + LR_{ind} \]
This joint test statistic has asymptotical distribution $\lambda^2(2)$. (Lopez 1999b.)

This initial independence test was later developed by Christoffersen and Pelletier (2004) to incorporate the assumption that if VaR violations exhibit complete independence, then the amount of time between two VaR breaks should be independent of the amount of time that passed since last break.

5.4.3 Regulatory loss functions

Underestimation of losses is important for supervisors while bank risk managers may be more concerned about higher VaR prediction values which lead to higher capital adequacy requirements. Lopez (1999b:7) suggested a regulatory loss function test in order to evaluate the accuracy of the VaR estimates. Unlike the previously revealed tests, this one does not involve hypothesis testing, but rather gives numerical measurement in accordance with specific regulatory concerns. This method provides comparative measure of performance through time and between different banks. The general form of this loss function of financial institution $i$ at time $t$ is:

\begin{equation}
C_{m_{t+1}} = \begin{cases} 
  f(\varepsilon_{t+1}, VaR_{mt}) & \text{if } \varepsilon_{t+1} < VaR_{mt} \\
  g(\varepsilon_{t+1}, VaR_{mt}) & \text{if } \varepsilon_{t+1} \geq VaR_{mt}
\end{cases}
\end{equation}

$\varepsilon_{t+1}$ represent the realized profit or loss, and $f(x, y)$ and $g(x, y)$ are functions that satisfy $f(x, y) \geq g(x, y)$. The numerical scores have negative orientation which means lower value of $C_{m_{t+1}}$ are better, since breaks are given higher score. The sum for the whole interval will be:

\begin{equation}
C_m = \sum_{i=1}^{T} C_{m_{t+1}}
\end{equation}
After loss function is defined and $C_m$ calculated, a benchmark can be calculated in order to evaluate the comparative performance of $VaR_{mt}$ estimates. Accurate VaR models should provide lower score than inaccurate ones. Different regulatory loss functions can be created. This work will utilize two different loss functions, a binary loss function (BLF) that takes into consideration whether at any given days the loss is greater or smaller than the VaR estimate, and a loss function adjusted for the change in the regulatory capital multiplier. (Lopez 1999:b.)

BLF counts whether the actual loss is larger or smaller than the VaR estimate. It is simply concerned about the number of failures. If $\varepsilon_{t+1} \geq VaR$, $g$ is given value of 1, it is a break, with all others events having a value of 0.

\[
C_{mt+1} = \begin{cases} 
0 & \text{if } \varepsilon_{t+1} < VaR_{mt} \\
1 & \text{if } \varepsilon_{t+1} \geq VaR_{mt} 
\end{cases}
\]
6 DATA DESCRIPTION

New EU member states present significant differences in the dynamics of financial markets, compared to older member’s economies. EU new member states experience higher financial turbulence, higher variability of liquidity, smaller trading volumes and shorter history of time series of returns. Since a considerable amount of capital from developed economies is invested in developing markets by banks, hedge and mutual funds, a careful investigation of the market dynamics in these economies would assist investors by increasing their awareness (Zikovic 2007:329). To answer the question which VaR models appropriately capture the market risk in the EU new member states, two VaR models are tested on the stock index. The tested VaR models are: Historical Simulation with rolling windows of 100 and 250 days, and Monte Carlo Simulation. For illustrating of rolling window technique take a window size of 250 days. The time interval is placed between the 1st and the 250th data points. Then VaR value forecast is obtained for the 251st day. The window is moved one period ahead to 2nd and 251st data points to obtain a forecast of the 252nd day return and so on for the whole period of observation.

Stock indices can be treated as a portfolio of selected securities from individual country. In this thesis, the performance of selected VaR models is tested on Bulgaria stock index (SOFIX) and compared with the behaviour of five world major market indices: S&P 500, FTSE 100, NASDAQ, Dow Jones, DAX, Stockholm General for matching periods of time. VaR values are calculated for a one-day holding period at 95% and 99% confidence intervals of risk. To be sure that the same out of the sample VaR backtesting period is used for all of the tested indexes, the data sets are matched on to the latest observations from each index. The rest of the observations are used as pre samples needed for VaR initial calculation. Analyzed VaR models validity is tested by Kupiec test, Christoffersen independence test, and Lopez test.

6.1 Overview of the Bulgarian stock exchange
Bulgarian Stock Exchange (BSE) was officially licensed by the state securities and exchange commission and started functioning on October 9, 1997. It is the country’s only operating stock exchange. Situated in Sofia, BSE operates within the Bulgarian legislation framework. It provides the execution of all trading activities in compliance with the Markets in Financial Instruments Act and the Law on public offering of securities. BSE rules and regulations are governed by BSE-Sofia Board of directors. Official regulator of the stock market is the state financial supervision commission. Its main purpose is to exercise control to safeguard the investors and to reinforce the evolution of a transparent and efficient securities market. The Commission is an independent state body and operates under the authority the Parliament. It regulates and controls: regulated securities markets, Central Depository, investment intermediaries, investment companies, management companies, natural persons who are directly engaged in securities transactions and investment consultancy, public companies and other issuers of securities according the Law on Public Offering of Securities and the Markets in Financial Instruments Act. (BSE 2007.)

The Commission acts as a controller over public companies and issuers. It is accountable for issues and withdraws of licenses and emits confirmations and public offering approvals. Its authorities audit the investment and depositories operation of banks and organize the information disclosure between the state institutions, self-governance administration and non-governmental organizations related to the securities market. (BSE 2007.)

6.2 SOFIX index

SOFIX is the first and most popular index of BSE-Sofia. Officially its calculation started on October 20, 2000. Its value is derived from the market capitalization of the shares of the twenty most liquid companies traded on the market. Daily adjustments are made for the free float (FF) of each share. FF represents the shares hold by minority shareholders which have no more than five percent of the votes at the company general meeting. In order for a company to be included in the index it should meet the following
criteria: Its shareholders should be no less than five hundred persons and the total value of trades for its issued shares during the last twelve months should be larger than 1,25 million €. The number of all transactions made in the same period must be bigger than one thousand. Also FF of the company should be greater than ten percent of the total amount of issued shares.

SOFIX daily value is calculated using the following methodology: the index base value (or the value from the previous trade day) is multiplied by the ratio of the sum of the market capitalisations of all issues for each company in the index portfolio as of the current moment and the sum of the market capitalization for the previous or base moment. Both of these sums are adjusted by a weight factor W, the free float FF and a divisor D. The market capitalization of each company is calculated as product of the number of the issued shares and the price taken from the newest deal on the market. For each daily individual company capitalization there is an upper cap limit of fifteen percent of the total daily SOFIX capitalization. If there are no deals for the shares of a company throughout the session, for the purpose of SOFIX calculation the last trading price from previous days is taken. The interval for index value update during a trading session is one minute. The Indices Committee meeting decides each quarter how much the weight and free-float factors shall be changed, taking into account their proportions for the new periods. (BSE 2007.)

The official formula for SOFIX calculation is as follow:

\[
SOFIX_t = SOFIX_{t-1} \times \left[ \frac{\sum_{i=1}^{n} N_{i,t} \times P_{i,t} \times FF_{i,t} \times W_{i,t} \times D_{i,t}}{\sum_{i=1}^{n} N_{i,t-1} \times P_{i,t-1} \times FF_{i,t-1} \times W_{i,t-1} \times D_{i,t-1}} \right] \times K
\]

Where \( N_{i,t} \) stands for the number of shares of the issue of the respective company on the day t, \( N_{i,t-1} \) is the number of shares of the issue of the respective company on the t-1 day, \( P_{i,t} \) is the price of the latest trade in the i-th security on the t day, \( P_{i,t-1} \) is the price of the last trade in the i-th security on the t-1 day, \( FF_{i,t} \) is the free-float of the i-th
security on the t day, $FF_{i,t-1}$ is the free-float of the i-th security on the t-1 day, $W_{i,t}$ is the weight factor of the i-th security on the t day, $W_{i,t-1}$ is the weight factor of the i-th security on the t-1 day, n is the number of issues included in the index portfolio, i is the indicator of the specific security, t is the day for which the index is calculated, $D_{i,t}$ is the divisor effective for the current trading session for the i-th security, while $D_{i,t-1}$ is the divisor for the i-th security on the t-1 day. K stands for the adjustment factor (K=1, unless the index base has been changed). Decisions for changing the base in the SOFIX base are taken every six months. (BSE 2007.)
7 EMPIRICAL RESULTS

Data set of daily returns price for ten years period was collected for the following market indices: SOFIX, S&P 500, NASDAQ, OMXS, FTSE 100 and DAX, in order to give representative overview of the developed world markets and compare them with the new EU member state Bulgaria. Bulgarian stock exchange, Google Finance and Yahoo Finance web pages are the sources of this data. The sample consists of daily closing prices for the period from 24 October 2000 to 30 April 2010, corresponding to 2367 observations. For SOFIX, the analyzed VaR models are Historical and Monte Carlo Simulations with rolling windows of 100 and 250 days, at 95% and 99% confidence level. For the other indices HS are performed with same time intervals and confidence levels.

7.1 Indices Returns

Descriptive statistics of indices daily returns are presented in Table 5. All means shown in column 1 are nearly zero, with SOFIX displaying the biggest positive diversion. This may be due to the higher inflation in Bulgaria. The average annual rate of inflation (consumer prices) in the country for the sample data period was 6.84%. Bulgarian stock market index also has the highest standard deviation (0.0188) of daily returns. According to the sample kurtosis figures, the daily rates of returns for SOFIX are far from being normally distributed. SOFIX kurtosis is over three times bigger than this of the other indices. This shows that the return distribution of Bulgarian markets have much fatter tail. The lowest kurtosis is exhibited by OMXS with value of 4.02. Column 4 of the descriptive statistic table shows that Indices returns have both positive and negative skewness. For SOFIX skewness is negative, indicating longer left tail, extended more towards negative values. SOFIX distribution appears to be left-skewed. Columns 6 and 7 of the table show the highest and lowest 1 day return for each index. The highest 1 day positive and negative returns are observed for SOFIX and NASDAQ. DAX and OMXS have lowest values in the category. The minimum and maximum statistics for SOFIX are comparatively large and that indicates the presence of extreme
returns. They can be observed in Figure 8-11 where returns are plotted. Plots exhibit extreme spikes.

Table 5. Indices Daily P&L Summary Statistics.

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>Range</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX</td>
<td>0.0006</td>
<td>0.0188</td>
<td>25.0685</td>
<td>-0.5648</td>
<td>0.4197</td>
<td>-0.2090</td>
<td>0.2107</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.0001</td>
<td>0.0139</td>
<td>8.3108</td>
<td>-0.1223</td>
<td>0.2043</td>
<td>-0.0947</td>
<td>0.1096</td>
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<td>NASDAQ</td>
<td>0.0000</td>
<td>0.0177</td>
<td>5.1678</td>
<td>0.1965</td>
<td>0.2284</td>
<td>-0.0959</td>
<td>0.1325</td>
</tr>
<tr>
<td>OMXS</td>
<td>0.0001</td>
<td>0.0149</td>
<td>4.0238</td>
<td>0.0355</td>
<td>0.1662</td>
<td>-0.0799</td>
<td>0.0863</td>
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<tr>
<td>FTSE 100</td>
<td>0.0001</td>
<td>0.0134</td>
<td>6.5040</td>
<td>-0.1126</td>
<td>0.1865</td>
<td>-0.0926</td>
<td>0.0938</td>
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<tr>
<td>DAX</td>
<td>0.0000</td>
<td>0.0167</td>
<td>4.4285</td>
<td>0.0643</td>
<td>0.1823</td>
<td>-0.0743</td>
<td>0.1080</td>
</tr>
</tbody>
</table>

Figure 6 presents QQ-plot of SOFIX returns compared to normal distribution. This plot also indicates that the series distribution is not normal. The QQ-plot does not follow flat diagonal line. SOFIX distribution is more skewed than Normal and has heavier tails and sharp peak.

![QQ Plot of Sample Data versus Standard Normal](image)

**Figure 6.** Quantile Quantile plotting of SOFIX Returns
**Table 6. Autocorrelations and Partial Autocorrelations of SOFIX Returns**

ACF, PACF and Ljung-Box Q test for mean adjusted returns and squared returns for SOFIX index in the period 24 October 2000 to 30 April 2010.

<table>
<thead>
<tr>
<th>Date: 06/17/12  Time: 19:29</th>
<th>Sample: 12337  Included observations: 2287</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autocorrelation</strong></td>
<td><strong>Partial Correlation</strong></td>
</tr>
<tr>
<td>1</td>
<td>-0.003</td>
</tr>
<tr>
<td>2</td>
<td>0.088</td>
</tr>
<tr>
<td>3</td>
<td>0.016</td>
</tr>
<tr>
<td>4</td>
<td>0.035</td>
</tr>
<tr>
<td>5</td>
<td>0.045</td>
</tr>
<tr>
<td>6</td>
<td>0.074</td>
</tr>
<tr>
<td>7</td>
<td>0.046</td>
</tr>
<tr>
<td>8</td>
<td>0.054</td>
</tr>
<tr>
<td>9</td>
<td>-0.003</td>
</tr>
<tr>
<td>10</td>
<td>0.054</td>
</tr>
<tr>
<td>11</td>
<td>0.025</td>
</tr>
<tr>
<td>12</td>
<td>-0.049</td>
</tr>
<tr>
<td>13</td>
<td>0.071</td>
</tr>
<tr>
<td>14</td>
<td>0.000</td>
</tr>
<tr>
<td>15</td>
<td>0.046</td>
</tr>
<tr>
<td>16</td>
<td>0.085</td>
</tr>
<tr>
<td>17</td>
<td>0.013</td>
</tr>
<tr>
<td>18</td>
<td>-0.023</td>
</tr>
<tr>
<td>19</td>
<td>-0.001</td>
</tr>
<tr>
<td>20</td>
<td>0.013</td>
</tr>
<tr>
<td>21</td>
<td>0.027</td>
</tr>
<tr>
<td>22</td>
<td>0.039</td>
</tr>
<tr>
<td>23</td>
<td>0.014</td>
</tr>
<tr>
<td>24</td>
<td>-0.019</td>
</tr>
<tr>
<td>25</td>
<td>0.028</td>
</tr>
<tr>
<td>26</td>
<td>0.032</td>
</tr>
<tr>
<td>27</td>
<td>-0.029</td>
</tr>
<tr>
<td>28</td>
<td>0.061</td>
</tr>
<tr>
<td>29</td>
<td>0.023</td>
</tr>
<tr>
<td>30</td>
<td>-0.009</td>
</tr>
<tr>
<td>31</td>
<td>0.045</td>
</tr>
<tr>
<td>32</td>
<td>0.001</td>
</tr>
<tr>
<td>33</td>
<td>-0.038</td>
</tr>
<tr>
<td>34</td>
<td>0.120</td>
</tr>
<tr>
<td>35</td>
<td>-0.037</td>
</tr>
<tr>
<td>36</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The presence of autocorrelation and heteroskedasticity in SOFIX returns is obvious from the ACF, PACF and Ljung-Box Q statistics of the returns and squared returns of SOFIX index in the corresponding correlograms (table 6). In the correlogram of the squared returns, the autocorrelations and partial autocorrelations are not zero for all lags. Such finding is troublesome for VaR models, based on normality assumption and for nonparametric approaches, which are based on the identically and independently distributed, iid, assumption, such as the historical simulation. Risk managers should be alert because VaR models elementary assumptions are not satisfied, meaning that VaR figures derived from such models cannot be completely trusted.

The obtained results so far show that normal distribution assumption poorly describes the fat tail and sharp peak characteristics of SOFIX index. In such cases Student's T distribution is proven to work better. To have a graphic comparison between returns
histogram, normal distribution, and T distribution, distribution fitting is presented in Figure 7. Student’s T distribution fits much closer to the realized historical returns.

Figure 7. SOFIX Daily Profit/Loss Distributions vs. Normal & Student’s T
Histograms of daily trading profit and loss reported from 24 October 2000 up to 30 April 2010 for SOFIX

Compared with the other indices, SOFIX exhibits asymmetry and leptokurtosis. It can be said with great certainty that its returns are not normally distributed and so we can reject the hypothesis of normal distribution.

7.2 VaR Results

The realized daily returns and SOFIX VaR values calculated by HS and MCS with data windows of 100 and 250 days for 95% and 99% confidence level are presented in figures 8 to 11. All other market Indices are shown in appendix 1. For the MC procedure, 250 random draws from normal distribution (with mean and standard deviation calculated from the previous observed real historical returns) were produced for each trading day throughout 10 years historical data. MCS VaR is calculated as percentile of these daily pseudo distributions. VaR violations, the situation when Loss is bigger than VaR level are clearly visible. The plot of log returns indicates volatility-
clustering phenomenon, large and small swings tend to cluster. From the figures it is clearly seen how the performance of different VaR models is affected by the length of the used data window. As expected, 100 days window is more responsive to changes in returns and the graph fits closer to the returns plot, while 250 days window displays lags and fewer variations after periods with big negative jumps in returns. HS and MCS VaR exhibits quite similar behaviour, using one and the same confidence level and data window. This can be explained by the fact that for MCS calculations, mean and standard deviation parameters are derived from the historic SOFIX returns.

Figure 8. Daily Profit and Loss and VaR 99% Forecasts with 250 days window
Time series of SOFIX daily trading profit and loss plotted with two model forecasts of the 1-day ahead 99th percentile P&L. The two models are HS and MCS VaR models using history of 250 trading days window.

Figure 9. Daily Profit and Loss and VaR 99% Forecasts with 100 days window
Figure 10. Daily Profit and Loss and VaR 95% Forecasts with 250 days window

Figure 11. Daily Profit and Loss and VaR 95% Forecasts with 100 days window
Table 7. Mean VaR values summary.

<table>
<thead>
<tr>
<th>Mean VaR</th>
<th>VaR 99% 250 days</th>
<th>VaR 99% 100 days</th>
<th>VaR 95% 250 days</th>
<th>VaR 95% 100 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX HS</td>
<td>-4.59%</td>
<td>-3.54%</td>
<td>-2.29%</td>
<td>-2.11%</td>
</tr>
<tr>
<td>SOFIX MCS</td>
<td>-3.49%</td>
<td>-3.56%</td>
<td>-2.50%</td>
<td>-2.55%</td>
</tr>
<tr>
<td>S&amp;P500 HS</td>
<td>-3.17%</td>
<td>-2.72%</td>
<td>-2.07%</td>
<td>-1.93%</td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>-3.54%</td>
<td>-3.09%</td>
<td>-2.51%</td>
<td>-2.30%</td>
</tr>
<tr>
<td>OMXS HS</td>
<td>-3.52%</td>
<td>-3.01%</td>
<td>-2.27%</td>
<td>-2.14%</td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td>-3.31%</td>
<td>-2.83%</td>
<td>-2.02%</td>
<td>-1.93%</td>
</tr>
<tr>
<td>DAX HS</td>
<td>-3.97%</td>
<td>-3.44%</td>
<td>-2.59%</td>
<td>-2.41%</td>
</tr>
</tbody>
</table>

Mean values for all calculated VaR models during the whole observation sample are presented in Table 7. As expected, 250 days data window produces higher VaR values for all HS. This finding is consistent with the regulatory use of 250 days for the calculation of capital reserves. The variation in the developed market indices is quite small, little over half a percent, while SOFIX mean is on average 1% higher. SOFIX and DAX exhibit the highest mean VaR values. MCS performance is contrary to that of HS and we witness higher VaR for 100 days observations. However, the difference is not big and varies with the simulation cycles.

Table 8 reports the actual violation rates for the VaR models, occurred during the whole sample period. With 2116 observations, at the 99% confidence level P&L would be expected to violate VaR around 21 times and for the 95% this level should be approximately 106. All HS VaR 99% confidence level results are significantly higher than the expectancy rate. For HS VaR 99%, calculated with 250 days interval, violations are less than these for HS VaR99% with 100 days interval. This fact indicates that overall, HS VaR 99% methods underestimate actual market risk level. This is a concerning finding for both regulators and risk managers. For HS VaR 95% breaks are still higher than the assumed level, though not as high as for 99%. HS VaR 95% level seems to match better the real risk and outperform VaR99% measure. These results justify VaR 95% popularity among risk practitioners. However, the same characteristic, less violation with 250 days window and more with 100 days interval is also observed for 95% confidence level. 250 days interval proves to be more reliable. Contradicting results were obtained from the MCS. While MCS VaR 99% produced worst results
compared to HS for both time intervals, at 95% confidence level MCS was the best performer and the only model with less then expected breaks level.

Table 8. VaR Violations number across indices with HS and MCS for SOFIX.
Daily VaR violations counted in matching sample of 2116 observations.

<table>
<thead>
<tr>
<th>VaR Violations number</th>
<th>VaR 99%</th>
<th>VaR 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>250 days</td>
<td>100 days</td>
</tr>
<tr>
<td>SOFIX HS</td>
<td>33</td>
<td>43</td>
</tr>
<tr>
<td>SOFIX MCS</td>
<td>58</td>
<td>53</td>
</tr>
<tr>
<td>S&amp;P500 HS</td>
<td>38</td>
<td>46</td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>31</td>
<td>38</td>
</tr>
<tr>
<td>OMXS HS</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>DAX HS</td>
<td>28</td>
<td>43</td>
</tr>
</tbody>
</table>

7.3 Correlation analysis

Table 9. Correlation of P&L across Indices.
Correlations calculated with matched sample of daily P&L with 2367 observations.

<table>
<thead>
<tr>
<th>Returns</th>
<th>SOFIX</th>
<th>S&amp;P 500</th>
<th>NASDAQ</th>
<th>OMXS</th>
<th>FTSE100</th>
<th>DAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.0141</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ</td>
<td>0.0227</td>
<td>0.9008</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMXS</td>
<td>0.0085</td>
<td>0.0252</td>
<td>0.0273</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTSE100</td>
<td>0.0264</td>
<td>0.0239</td>
<td>0.0464</td>
<td>-0.0124</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>DAX</td>
<td>-0.0438</td>
<td>0.0202</td>
<td>0.0207</td>
<td>-0.0282</td>
<td>0.0218</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Returns correlation coefficients across indices show no significant correlation except for S&P 500 and NASDAQ, which is normal as they both represent US market. DAX, OMXS and FTSE 100 exhibit small negative correlation values. Low return correlations reflect the differences in market compositions across various countries.
Tables 10 to 13 display correlations between the different HS and MCS VaR models. All VaR results show high positive correlation. The model with 95% confidence level and 250 days window displayed the highest results. Generally 99% coverage VaR displayed lower correlation than 95%, and VaR models with 100 days events window were less correlated compared to 250 days. In all correlation tables HS and MCS VaR for SOFIX have values higher than 0.9 which represents the fact that major parameters for MCS were estimated from historical distribution. These findings are consistent with the observed speed with which the recent financial crisis spread over the Global Financial Market and are in contrast with the insignificant daily cross-correlations in P&L.

**Table 10.** Correlations of VaR 99% 250 days Across Indices.
Correlations calculated with matched sample of daily VaR estimates with 2116 observations.

<table>
<thead>
<tr>
<th>VaR 99% 250 days</th>
<th>SOFIX HS</th>
<th>SOFIX MCS</th>
<th>S&amp;P 500 HS</th>
<th>NASDAQ HS</th>
<th>OMXS HS</th>
<th>FTSE100 HS</th>
<th>DAX HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX HS</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOFIX</td>
<td></td>
<td>0.95993</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCS</td>
<td>0.71764</td>
<td>0.69211</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 HS</td>
<td>0.87601</td>
<td>0.85322</td>
<td>0.932345</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>0.58503</td>
<td>0.57234</td>
<td>0.898346</td>
<td>0.83562</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMXS HS</td>
<td>0.71787</td>
<td>0.69671</td>
<td>0.918214</td>
<td>0.88221</td>
<td>0.90502</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td>0.79372</td>
<td>0.80083</td>
<td>0.849353</td>
<td>0.89707</td>
<td>0.787902</td>
<td>0.92933</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 11. Correlations of VaR 95% 250 days Across Indices.

<table>
<thead>
<tr>
<th>VaR 95% 250 days</th>
<th>SOFIX HS</th>
<th>SOFIX MCS</th>
<th>S&amp;P 500 HS</th>
<th>NASDAQ HS</th>
<th>OMXS HS</th>
<th>FTSE100 HS</th>
<th>DAX HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX HS</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOFIX</td>
<td></td>
<td>0.92095</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCS</td>
<td>0.90061</td>
<td>1</td>
<td>0.75474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 HS</td>
<td></td>
<td></td>
<td>0.90061</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>0.92091</td>
<td>0.89991</td>
<td>0.91677</td>
<td>0.75474</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMXS HS</td>
<td>0.82948</td>
<td>0.71992</td>
<td>0.94834</td>
<td>0.89787</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.89787</td>
<td>1</td>
</tr>
<tr>
<td>DAX HS</td>
<td>0.72511</td>
<td>0.69688</td>
<td>0.81065</td>
<td>0.86316</td>
<td>0.8457</td>
<td>0.91032</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 12. Correlations of VaR 99% 100 days Across Indices.

<table>
<thead>
<tr>
<th>VaR 99% 100 days</th>
<th>SOFIX HS</th>
<th>SOFIX MCS</th>
<th>S&amp;P 500 HS</th>
<th>NASDAQ HS</th>
<th>OMXS HS</th>
<th>FTSE100 HS</th>
<th>DAX HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX HS</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOFIX</td>
<td></td>
<td>0.90913</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCS</td>
<td>0.45239</td>
<td>0.60773</td>
<td>0.90913</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 HS</td>
<td></td>
<td></td>
<td>0.45239</td>
<td>0.60773</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>0.71068</td>
<td>0.79323</td>
<td>0.91317</td>
<td>0.60773</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMXS HS</td>
<td>0.35568</td>
<td>0.45574</td>
<td>0.8079</td>
<td>0.79323</td>
<td>0.91317</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8079</td>
<td>0.91317</td>
</tr>
<tr>
<td>DAX HS</td>
<td>0.43415</td>
<td>0.56214</td>
<td>0.92474</td>
<td>0.8079</td>
<td>0.79323</td>
<td>0.91317</td>
<td>1</td>
</tr>
</tbody>
</table>

DAX HS

DAX HS
Table 13. Correlations of VaR 95% 100 days Across Indices.

<table>
<thead>
<tr>
<th>VaR 95% 100 days</th>
<th>SOFIX HS</th>
<th>SOFIX MCS</th>
<th>S&amp;P 500 HS</th>
<th>NASDAQ HS</th>
<th>OMXS HS</th>
<th>FTSE100 HS</th>
<th>DAX HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFIX HS</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOFIX MCS</td>
<td>0.9279</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 HS</td>
<td>0.7151</td>
<td>0.63805</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ HS</td>
<td>0.81695</td>
<td>0.83741</td>
<td>0.86267</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMXS HS</td>
<td>0.63866</td>
<td>0.57216</td>
<td>0.90987</td>
<td>0.81011</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTSE100 HS</td>
<td>0.62504</td>
<td>0.5908</td>
<td>0.91473</td>
<td>0.81504</td>
<td>0.90873</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DAX HS</td>
<td>0.57124</td>
<td>0.5874</td>
<td>0.82019</td>
<td>0.78588</td>
<td>0.82627</td>
<td>0.91401</td>
<td>1</td>
</tr>
</tbody>
</table>

7.4 Backtesting results

Backtesting procedure was carried out in order to validate the accuracy of the HS and MCS 99 % VaR values for 250 days interval for SOFIX return. This choice is based on the official Basel II regulatory reporting standard. Returns from year 2000 were used to generate initial VaR values. Altogether nine years of market data are presented in table 14. Reported values are taken from the last trading day of each year. Column 2 present the sum of yearly breaks, column 3 LR un is the likelihood ratio value and p value indicates the probability of the actual violation rate if the null of 1 % expect rate is true. The null hypothesis of Kupiec’s unconditional coverage backtest is that daily P&L will violate VaR at an expected rate of 1 %.
Table 14. Kupiec’s test results for unconditional coverage of HS & MCS VaR 99% 250 days for SOFIX index.

The presented values are taken for the last trading day of each Year. The p-value is computed in Excel using the cumulative $\chi^2$-distribution with one degree of freedom $P = CHI2DIST (LR; 1)$.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sum of breaks</th>
<th>LR un</th>
<th>P value</th>
<th>Sum of breaks</th>
<th>LR un</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0</td>
<td>na</td>
<td>na</td>
<td>0</td>
<td>na</td>
<td>na</td>
</tr>
<tr>
<td>2008</td>
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The Backtest results are compared in line with Basel II framework for the minimal capital requirements multiplication factor. As previously described, the multiplier for capital requirements begins to rise in the yellow zone after four VaR violations in the previous 250 days. This threshold equals to a value of 0.33 for the Kupiec’s POF test. The threshold for the red zone with ten VaR violations in which the multiplier is set to its maximal value of 4.0 (the underlying VaR model is deemed inaccurate) is equal to a POF test value of 5.6. Each reported result is less likely, if it have lower p-value, in case the null hypothesis is true. The reported p-values are at 1% of significance, correspondingly the null hypothesis is rejected if p is less then 0,01. For years 2007 and 2008, both simulations fail the unconditional coverage test. MCS has almost twice higher level of breaks in 2008, compared with HS. This finding confirms the previous conclusion that based on the Normal distribution assumption MCS will perform badly in high volatile markets. MCS also produced twice higher break number compared with HS in year 2005, though both models still passed Kupiec’s test. All other investigated periods produced correct unconditional coverage. It have to be pointed out that year 2000 had extremely high volatility level, with the initial establishment of the Bulgarian stock exchange. This historical interval produced high VaR estimates that were not
broken in the following 2001 even once. It is the same case in years 2003 and 2009. MCS however did produce a break in year 2003.

To further explore the behaviour of VaR measures, SOFIX market return’s volatility and Lopez’s Binary Loss Function were plotted in Figures 12 to 15 for HS and MCS with 95% and 99% coverage for both time windows (for the other indices HS see plots in appendix 1). An intuitive property of a VaR (since the VaR is a quantile) is that it is positively related to volatility. Under some distribution assumptions for the revenues, as Jorion (2000) showed, the relationship between VaR and volatility should be linear. Results for SOFIX display such behaviour for 95% 250 days model. This model produced almost identical BLF for the two simulations. Generally, both HS & MCS exhibit similar behaviour for 250 days period, while for 100 the differences between them become obvious. In all plots MCS VaR deviates stronger from the expected level of breaks than HS. This observation seems due to the normal distribution assumption implemented in the MCS and the low number of simulations loops preformed because of computational power limitations and long historical return data set. For the 250 window under both percentages of confidence, we observe persistent drops in VaR break values in periods when market volatility is in steep decline. Obviously in such time intervals 250 days VaR models over predict market risk. In fact the highest VaR breaks levels are observed during steep volatility jumps, while even when volatility levels are high, if no big jumps are present, VaR break level goes down. VaR with 100 days data window produces not such clear pattern of behaviour, with both HS and MCS varying regularly above and below expectance level. Still, MCS again exhibit higher level of breaks for 95% coverage. It is clear that shorter observation window is reflecting better the changes in volatility, and according to BLF results, HS for SOFIX outperformed MCS.
Figure 12. Binary Loss Function plot for VaR 99% 250 days window.
1% expectance level is shown as flat black line.

Figure 13. Binary Loss Function plot for VaR 99% 100 days window.
Figure 14. Binary Loss Function plot for VaR 95 % 250 days window.
5% expectance level is shown as flat black line.

Figure 15. Binary Loss Function plot for VaR 95 % 100 days window.
8 CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Employed tests over the distribution of daily returns of SOFIX index showed that it exhibit substantial differences from the developed financial markets worldwide. Bulgarian Stock exchange returns are characterized by significant asymmetry and high kurtosis that leads to rejection of the hypothesis of normal distribution. Autocorrelation was found in the squared returns of SOFIX index. This phenomenon breaches the normality assumption, as well as the “identical and independent distributed” assumption, which is a necessary requirement for the appropriate implementation of HS. As elementary model postulates is not satisfied, the derived VaR figures cannot be completely trusted.

Both of the performed simulations in this study, HS and MCS, provided satisfactory unconditional coverage for the period 2001-2006. In times of low market volatility HS VaR 99% models with 250 days data window over predict market risk. For the volatile period of the recent market crisis 2007-2009 none of the models passed the unconditional coverage test. The highest VaR breaks levels were observed during steep volatility jumps which indicate that VaR reacts poorly to volatility changes and underestimate risk in such conditions. This finding confirms study Hypothesis III.

According to BLF results, HS with 99% confidence level for SOFIX outperformed MCS, and 250 days interval proves to be more reliable than 100 days. However, HS uses only realized past returns to calculate VaR and these returns in the observed window have a crucial influence on the acceptability of HS model, consequently on MCS models based on historical values. Depending only on past realized market returns can give a deteriorated estimate of true level of risk. Results lead to the conclusion that even though HS provided correct unconditional coverage for most of the observed time, use of only HS (particularly based on short observation periods) is not recommendable as regulatory capital requirement criteria in new EU markets.
Based on the backtesting results it can be derived that VaR models that are commonly used in developed stock markets are not well suited for measuring market risk in EU new member states in time of severe market crisis. Although there is a common belief that more information is better, for investors, creditors, and other users of VaR information, primarily concern should be the accuracy of measures. It seems not enough to simply implement ready VaR models offered by software providers. Regulators have to be particularly concerned with simplistic VaR models that are popular in developed countries. They are not well suited for illiquid and developing stock markets. This makes proper VaR estimation more complicated and requires complex computational and intelligence demanding VaR models. For such reasons, before allowing banks in new EU member states to use internal VaR models, regulators should analyse the backtesting performance and the theoretical framework of any model for inconsistencies or redundant simplifications.
REFERENCES


APPENDIX 1. RETURN PLOTS & VAR VALUE FOR S&P 500, NASDAQ, OMXS, FTSE 100 AND DAX.